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A NOTE ON SOFT ROUGH MINIMAL AND SOFT ROUGH MAXIMAL OPEN SETS

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ABSTRACT
The notions of some theorems on soft rough minimal open set, soft rough maximal open set and also soft rough minimal irresolute and soft rough maximal irresolute are introduced and studied. We also investigate some related properties of these concepts.

Keywords: Soft rough minimal open set, soft rough maximal open set, soft rough minimal irresolute, soft rough maximal irresolute.

1. INTRODUCTION

Rough set was introduced by Pawlak [1] for dealing with vagueness and granularity in information systems. This theory deals with the approximation of an arbitrary subset of a universe by two definable or observable subsets called lower and upper approximation. The reference space in rough set theory is the approximation space whose topology is generated by the equivalence classes of R. Rough set theory was proposed as a new approach to processing of incomplete data. One of the aims of the rough set theory is a description of imprecise concepts. Suppose we are given a finite non empty set U of objects called universe. Each object of U is characterized by a description, for example a set of attributes values. In rough set theory an equivalence relation (reflexive, symmetric, and transitive relation) on the universe of objects is defined based on their attribute values. In particular, this equivalence relation constructed based on the equality relation on attribute values.

In the year 1999, Russian specialist Molodtsov[2], started the idea on soft sets as another scientific instrument to manage vulnerabilities while demonstrating issues in building material sciences, software engineering, financial aspects, sociologies and restorative sciences as a general mathematical tool for dealing with uncertain objects. To solve complicated problems in economics, engineering, environmental science and social science, methods in classical mathematics are not always successful because of various types of uncertainties presented in these problems. While probability theory, fuzzy set theory [4], rough set theory [1,5], and other mathematical tools are well known and often useful approaches to describing uncertainty, each of these theories has its inherent difficulties as pointed out by Molodtsov in [2,6]. The concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainties. This so-called soft set theory is free from the difficulties affecting existing methods.

Feng Feng et., al [3] used soft set theory to generalize Pawlak rough set model. Based on the novel granulation structures they introduced soft approximation spaces, soft rough approximation and soft rough sets and studied some of the properties of soft approximation spaces and the same properties of Pawlak approximation space.

Presently, works on soft set theory are progressing rapidly. Maji et al. [7] defined several operations on soft sets and made a theoretical study on the theory of soft sets. Aktas and Cagman [8] compared soft sets to the related concepts of rough sets. M.Lellies Thivagar et.,al [9] introduced rough topology and Muhammad Shabir, et.,al [10] introduced soft topological spaces.

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2. PRILIMINARIES

Definition 2.1 [2]: Let A be a subset of E. A $pair\ (F, A)$ is called the Soft Set over X. Where $F : A \rightarrow P(X)\ defined\ by\ F(e) \rightarrow P(X)\ for\ all\ e\ \in A.$ In other words, $F(e)$ may be considered as the set of $e - approximate\ element$ of the soft set (F, A).

Definition 2.2 [1]: Suppose U be a finite nonempty set called the Universe and R be an equivalence relation on U. The pair (U, R) is called the approximation space. Let X be a subset of U and $R(x)$ denoted the equivalence class determined by x.

i) The lower approximation space of a subset $X\ of\ U$ is defined as:

$$\underline{R}(X) = \cup_{x \in U}\ R(x) : R(x) \subseteq U$$

ii) The upper approximation space of a subset $X\ of\ U$ is defined as:

$$\overline{R}(X) = \cup_{x \in U}\ R(x): R(x) \cap X \neq \emptyset$$

iii) The boundary region of X with respect to R is defined as:

$$B_R(X) = \overline{R}(X) - \underline{R}(X)$$

The set X is said to be rough with respect to R, if $\overline{R}(X) \neq \underline{R}(X)$, i. e., $B_R(X) \neq \emptyset$

Definition 2.3: [10] Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if

1) $\emptyset\ and\ \tilde{X}$ belong to τ
2) The union of any number of soft sets in τ belong to τ
3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.4: [9] Let U be the universe of objects, R be an equivalence relation on U and $\tau_R = \{U, \emptyset, R_*(X), R^*(X), B_R(X)\}$ where $X \subseteq U$. τ_R satisfies the following axioms:

i) $U\ and\ \emptyset$ belong to τ
ii) The union of elements of any sub collection of τ_R is in τ_R
iii) The intersection of the elements of any finite sub collection of τ_R is in τ_R

τ_R forms a topology on U called as a rough topology on U with respect to X. We call (U, τ_R, X) as a rough topological space.

Definition 2.5: [11] Let $\mathfrak{S} = (f, A)$ be a Soft set over U. The pair $S = (U, \mathfrak{S})$ is called a Soft approximation space. Based on S, we define the following two operators:

$$\underline{apr_s}(X) = \{u \in U; \exists a \in A\ (u \in f(a) \subseteq X)\},$$
$$\overline{apr_s}(X) = \{u \in U; \exists a \in A(u \in f(a), f(a) \cap X \neq \emptyset)\},$$

assigning to every subset X $\subseteq$ U two sets $\underline{apr_s}(X)$ and $\overline{apr_s}(X)$ called the lower and upper soft rough approximations of X in S , respectively. If $\underline{apr_s}(X) = \overline{apr_s}(X)$, X is said to be soft definable; Otherwise X is called a soft rough set.

Clearly, $\overline{apr_s}(X)$ and $\underline{apr_s}(X)$ can be expressed equivalently as:

$\underline{apr_s}(X) = \cup_{a \in A} \{f(a); f(a) \subseteq X\}$,
$\overline{apr_s}(X) = \cup_{a \in A} \{f(a); f(a) \cap X \neq \emptyset\}$.

Definition 2.6: [12] Let U be the Universe, $P = (U, S)$ be a soft approximation space and $\tau_{SR}(X) = \left\{U, \emptyset, \underline{R_P}(X), \overline{R_P}(X), Bnd(X)\right\}$, Where $X \subseteq U$. $\tau_{SR}(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_{SR}(X)$,
(ii) The union of the elements of any sub collection of $\tau_{SR}(X)$ is in $\tau_{SR}(X)$,
(iii) The intersection of the elements of any finite sub collection of $\tau_{SR}(X)$ is in $\tau_{SR}(X)$.

$\tau_{SR}(X)$ forms a topology on U called as the soft rough topology on U with respect to X. $(U, \tau_{SR}(X), E)$ is called a Soft Rough Topological Space.

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Definition 2.7: [12] Let $(U, \tau_{SR}(X), E)$ be a soft rough topological space with respect to X, where $X \subseteq U$ and $A \subseteq U$. The soft rough interior of A is defined as the union of all soft rough open subsets of A and it is denoted by $SR - int(A)$. The soft rough closure of A is defined as the intersection of all soft rough closed subsets containing A and it is denoted by $SR - Cl(A)$.

Definition 2.8: [12] Let $(U, \tau_{SR}(X), E)$ and $(V, \tau_{SR'}(Y), E)$ be two soft rough topological spaces with respect to X and Y respectively. $\tau_{SR'}(Y)$ is finer than $\tau_{SR}(X)$, if $\tau_{SR'}(Y) \supseteq \tau_{SR}(X)$.

Definition 2.9: [13] A proper nonempty open subset U of a topological space X is said to be a minimal open set if any open set which is contained in U is $\emptyset$ or U.

Definition 2.10: [14] A proper nonempty open subset U of a topological space X is said to be a maximal open set if any open set which is contains U is X or U.

Definition 2.11: [15] A proper nonempty closed subset F of a topological space X is said to be a minimal closed set if any closed set which is contained in F is $\emptyset$ or F.

Definition 2.12: [15] A proper nonempty closed subset F of a topological space X is said to be a maximal closed set if any closed set which contains in F is X or F.

Theorem 2.13: [15] Let X be a topological space and $F \subset X$. F is minimal closed set if and only if $X - F$ is maximal open set.

Theorem 2.14 [15]: Let X be a topological space and $U \subset X$. U is a minimal open set if and only if $X - U$ is maximal closed set.

Definition 2.15 [16]: Let X and Y be the topological spaces. A map $f: X \to Y$ is called minimal irresolute if $f^{-1}(M)$ is minimal open set in X for every maximal open set M in Y.

Definition 2.16 [16]: Let X and Y be the topological spaces. A map $f: X \to Y$ is called maximal irresolute if $f^{-1}(M)$ is maximal open set in X for every maximal open set M in Y.

Definition 2.17 [17]: A proper nonempty soft open subset F_K of a soft topological space $(F_A, \tilde{\tau})$ is said to be minimal soft open set if any soft open set which is contained in F_K is $F_\emptyset$ or F_K

Definition 2.18 [17]: A proper nonempty soft open subset F_K of a soft topological space $(F_A, \tilde{\tau})$ is said to be maximal soft open set if any soft open set which contains F_K is F_A or F_K.

Definition 2.19 [17]: A proper nonempty soft closed subset F_C of a soft topological space $(F_A, \tilde{\tau})$ is said to be minimal soft closed set if any soft closed set which is contained in F_C is $F_\emptyset$ or F_C.

Definition 2.20 [17]: A proper nonempty soft closed subset F_C of a soft topological space $(F_A, \tilde{\tau})$ is said to be maximal soft closed set if any soft closed set which contains F_C is F_A or F_C.

Definition 2.21 [18]: Let $(U, \tau R(X))$ be a rough topological space. A non empty rough closed (resp. rough open) set A of U is said to be a rough minimal closed (resp. rough minimal open) set if only if any rough closed (resp. rough open) set which is contained in A is $\emptyset$ or A.

Definition 2.22:[18] Let $(U, \tau R(X))$ be a rough topological space. A non empty rough closed (resp. rough open) set A of U is said to be a rough maximal closed (resp. rough maximal open) set if only if any R closed (resp. R open) set which is contain in A is $\emptyset$ or U.

Definition 2.23:[19] A proper nonempty soft rough open subset $\tau_{SR}(x)$, of a soft rough topological space $(U, \tau_{SR}(X), E)$ is said to be soft rough minimal open set if any soft rough open set which is contained in $\tau_{SR}(x)$ is $\emptyset$ or $\tau_{SR}(x)$. The family of all soft rough minimal open sets in a soft rough topological space $(U, \tau_{SR}(X), E)$ is denoted by $SR\ Mi\ O(\tau_{SR}(X))$

Definition 2.24:[19]A proper nonempty soft rough open subset $\tau_{SR}(x)$ of a soft rough topological space $(U, \tau_{SR}(X), E)$ is said to be soft rough maximal open set if any soft rough open set which contains $\tau_{SR}(x)$ is U or $\tau_{SR}(x)$. The family of all soft rough maximal open sets in a soft rough topological space $(U, \tau_{SR}(X), E)$ is denoted by $SR\ Ma\ O(\tau_{SR}(X))$.

Definition 2.25: [19]Let $(U, \tau_{SR}(X), E)$ and $(V, \tau_{SR}(Y), E)$ be the soft rough topological spaces .The mapping $f: (U, \tau_{SR}(X), E) \rightarrow (V, \tau_{SR}(Y), E)$ is called soft rough minimal continuous if
$f^{-1}(F(E))$ is a soft rough open set in $(U, \tau_{SR}(X), E)$ for each soft rough minimal open set $F(E)$ in $(V, \tau_{SR}(Y), E)$.

Definition 2.26: [19] Let $(U, \tau_{SR}(X), E)$ and $(V, \tau_{SR}(Y), E)$ be the soft rough topological spaces. The map $f: (U, \tau_{SR}(X), E) \rightarrow (V, \tau_{SR}(Y), E)$ is called the soft rough maximal continuous if $f^{-1}(F(E))$ is a soft rough open set in $(U, \tau_{SR}(X), E)$ for each soft rough maximal open set $F(E)$ in$(V, \tau_{SR}(Y), E)$.

3. SOFT ROUGH MAXIMAL OPEN SETS AND SOFT ROUGH MINIMAL OPEN SET:

Let U be an initial universe, $X \subset U$, $P(U)$ be the set of all subsets of U, E be the set of parameters, $A \subset E$, $P = (U, S)$ be the approximation space in U and $(U, \tau_{SR}(X), E)$ be the soft rough topological space with parameter E. $\underline{R_P}(X)$ is lower limit, $\overline{R_P}(X)$ is upper limit and $Bnd(X)$ is boundary of soft rough topological space.

Lemma 3.1: Let (U, $\tau_{SR}(X), E)$ is a soft rough topological space.

i) Let $\tau_{SR}(x)$ be a soft rough maximal open set and $\tau_{SR}(y)$ is a soft rough open set in U with respect to X, then $\tau_{SR}(x) \cup \tau_{SR}(y) = U$ or $\tau_{SR}(y) \subset \tau_{SR}(x)$.

ii) Let $\tau_{SR}(x)\ and\ \tau_{SR}(z)$ be the soft rough maximal open sets. Then $\tau_{SR}(x) \cup \tau_{SR}(z) = U$ or $\tau_{SR}(x) = \tau_{SR}(z)$.

Proposition 3.2: Let $\tau_{SR}(x)$ be a soft rough maximal open set. If $\{x\}$ is an element of $\tau_{SR}(x)$, then for any soft rough open neighborhood $\tau_{SR}(n)$ of $\{x\}$, $\tau_{SR}(n) \cup \tau_{SR}(x) = U$ or $\tau_{SR}(n) \subset \tau_{SR}(x)$.

Proof: By lemma 3.1(i), the result follows.

Proposition 3.3: Let $\tau_{SR}(x)$ be a soft rough maximal open set and $\{x\}$ be an element of U with respect to X,
Then,$\tau_{SR}(x) = \cup\{\tau_{SR}(n): \tau_{SR}(n) \cup \tau_{SR}(x) \neq U$,where $\tau_{SR}(n)$ is a soft rough neighborhood of{x}}.

Proof: Since $\tau_{SR}(x)$ is an open neighborhood of $\{x\}$ and by proposition3.2, $\tau_{SR}(x) \subset \cup \{\tau_{SR}(n): \tau_{SR}(n) \cup \tau_{SR}(x) \neq U, where\ \tau_{SR}(x)\ is\ a\ soft\ rough\ neighborhood\ of\ \{x\}\} \subset \tau_{SR}(x)$.
Thus,$\tau_{SR}(x) = \cup\{\tau_{SR}(n): \tau_{SR}(n) \cup \tau_{SR}(x) \neq U$,where $\tau_{SR}(n)$ is a soft rough neighborhood of{x}}.

Theorem: 3.4: Let $\tau_{SR}(x)$ be soft rough maximal open set and $\{x\}$ be an element of $U - \tau_{SR}(x)$. Then, $U - \tau_{SR}(x) \subset \tau_{SR}(n)$ for any open neighborhood $\tau_{SR}(n)\ of\ \{x\}$.

Proof: Given $\{x\} \in U - \tau_{SR}(x)$ and we have $\tau_{SR}(n) \not\subset \tau_{SR}(x)$ for any soft rough open neighborhood $\tau_{SR}(n)\ of\ \{x\}$. then by lemma 3.1(i), $\tau_{SR}(n) \cup \tau_{SR}(x) = U$. Thus, we have $U - \tau_{SR}(x) \subset \tau_{SR}(n)$.

Corollary 3.5: Let $\tau_{SR}(x)$ be a soft rough maximal open set. Then, either of the following holds

i) For each $\{x\} \in U - \tau_{SR}(x)$ and each soft rough open neighborhood $\tau_{SR}(n)$ of $\{x\}$, $\tau_{SR}(n) =$ U.

ii) There exists a soft rough open set $\tau_{SR}(n)$ such that $U - \tau_{SR}(x) \subset \tau_{SR}(n)$ and $\tau_{SR}(n) \subsetneq$ U.

Proof: Consider (i) does not hold, then there exists an element $\{x\}$ of $U - \tau_{SR}(x)$ and soft rough open neighborhood $\tau_{SR}(n)$ of $\{x\}$ such that $\tau_{SR}(n) \subsetneq U$. By theorem 3.4, we have $U - \tau_{SR}(x) \subset \tau_{SR}(n)$.

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Corollary 3.6: Let $\tau_{SR}(x)$ be a soft rough maximal open set. Then, either of the following holds

i) For each $\{x\} \in U - \tau_{SR}(x)$ and each soft rough open neighborhood $\tau_{SR}(n)$ of $\{x\}$, $U - \tau_{SR}(x) \subsetneq \tau_{SR}(n)$.

ii) There exists a soft rough open set $\tau_{SR}(n)$ such that $U - \tau_{SR}(x) = \tau_{SR}(n) \neq U$.

Proof: Consider (ii) does not hold, then by theorem 3.4 $U - \tau_{SR}(x) \subset \tau_{SR}(n)$ for each $\{x\} \in U - \tau_{SR}(x)$ and soft rough open neighborhood $\tau_{SR}(n)$ of $\{x\}$. Hence, we have $U - \tau_{SR}(x) \subsetneq \tau_{SR}(n)$.

Theorem 3.7: Let $\tau_{SR}(x)$ be a soft rough maximal open set. Then,

i) $SR\ cl(\tau_{SR}(x)) = U$ or $SR\ cl\ (\tau_{SR}(x)) = \tau_{SR}(x)$.

ii) $SR\ int\ (U - \tau_{SR}(x)) = U - \tau_{SR}(x)$ or $SR\ int\ (U - \tau_{SR}(x)) = \emptyset$

Proof:

i) Given $\tau_{SR}(x)$ is a soft rough maximal open set, by corollary 3.6 we have two cases.

Case 1): Consider for each $\{x\} \in U - \tau_{SR}(x)$ and each soft rough open neighborhood $\tau_{SR}(n)$ of $\{x\}$. we have, $\tau_{SR}(n) \cap \tau_{SR}(x) \neq \emptyset$ for any soft rough neighborhood $\tau_{SR}(n)$ of $\{x\}$ by $U - \tau_{SR}(x) \neq \tau_{SR}(n)$. Therefore, $U - \tau_{SR}(x) \subset SR\ cl\ (\tau_{SR}(x))$.Since $U = \tau_{SR}(x) \cup \left(U - \tau_{SR}(x)\right) \subset \tau_{SR}(x) \cup SR\ cl\ \left(\tau_{SR}(x)\right) = SR\ cl\ \left(\tau_{SR}(x)\right) \subset U$, we have, $SR\ cl\ (\tau_{SR}(x)) = U$.

Case 2): Consider there exists a soft rough open set $\tau_{SR}(n)$ such that $U - \tau_{SR}(x) = \tau_{SR}(n) \neq U$. Since $U - \tau_{SR}(x) = \tau_{SR}(n)$ is a soft rough open set, $\tau_{SR}(x)$ is a soft rough closed set. Hence, $SR\ cl\ (\tau_{SR}(x)) = \tau_{SR}(x)$.

ii) The proof is obvious by corollary 3.6.

Theorem 3.8: Let $\tau_{SR}(x)$ be a soft rough maximal open set and $\tau_{SR}(l)$ a non empty soft rough open subset of $U - \tau_{SR}(x)$. Then, $SR\ cl\ (\tau_{SR}(l)) = U - \tau_{SR}(x)$.

Proof: Given $\tau_{SR}(l) \subset (U - \tau_{SR}(x))$ and it is non empty. Therefore by theorem 3.4 , we have $\tau_{SR}(n) \cap \tau_{SR}(l) \neq \emptyset$ for any element $\{x\}$ of $U - \tau_{SR}(x)$ and any soft rough open neighborhood $\tau_{SR}(n)$ of $\{x\}$. Then, $U - \tau_{SR}(x) \subset SR\ cl\ (\tau_{SR}(l))$. Since, $U - \tau_{SR}(x)$ is a closed set and $\tau_{SR}(l) \subset (U - \tau_{SR}(x))$, we have $SR\ cl\ \left(\tau_{SR}(l)\right) \subset SR\ cl\ \left(U - \tau_{SR}(x)\right) = U - \tau_{SR}(x)$. Therefore, $SR\ cl(\tau_{SR}(l)) = U - \tau_{SR}(x)$.

Corollary 3.9: Let $\tau_{SR}(x)$ be a soft rough maximal open set and $\tau_{SR}(m)$ a soft rough subset of U with $\tau_{SR}(x) \subsetneq \tau_{SR}(m)$. Then, $SR\ cl\ (\tau_{SR}(m)) = U$.

Proof: Since $\tau_{SR}(x) \subsetneq \tau_{SR}(m) \subset U$, there exists a nonempty soft rough subset $\tau_{SR}(l)$ of $U - \tau_{SR}(x)$ such that $\tau_{SR}(m) = \tau_{SR}(x) \cup \tau_{SR}(l)$. Hence, by theorem 3.8 $SR\ cl\ \left(\tau_{SR}(m)\right) = SR\ cl\ \left(\tau_{SR}(l) \cup \tau_{SR}(x)\right) = \mathrm{SR\ cl}\ \left(\tau_{SR}(l)\right) \cup SRcl\ \left(\tau_{SR}(x)\right) \supset \left(\mathrm{U} - \tau_{SR}(x)\right) \cup \tau_{SR}(x) = \mathrm{U}$. Therefore, SR cl $(\tau_{SR}(m)) = \mathrm{U}$.

Theorem 3.10: Let $\tau_{SR}(x)$ be a soft rough maximal open set and assume that there exists at least two elements in the subset $U - \tau_{SR}(x)$. Then, $SR\ cl\ (U - \{a\}) = U$for any element $\{\mathrm{a}\}$ of $U - \tau_{SR}(x)$.

Proof: The proof is obvious by corollary 3.9.

Theorem 3.11: Let $\tau_{SR}(x)$ be a soft rough maximal open set and $\tau_{SR}(c)$ a soft rough proper subset of U with $\tau_{SR}(x) \subset \tau_{SR}(c)$. Then, $SR\ int\ \left(\tau_{SR}(c)\right) = \tau_{SR}(x)$.

Proof: case 1): When $\tau_{SR}(c) = \tau_{SR}(x)$, we have $SR\ int\ \left(\tau_{SR}(c)\right) = SR\ int\ \left(\tau_{SR}(x)\right) = \tau_{SR}(x)$.

Case 2): When $\tau_{SR}(c) \neq \tau_{SR}(x)$, here $\tau_{SR}(x) \subsetneq \tau_{SR}(c) \Rightarrow \tau_{SR}(x) \subset \mathrm{SR\ int}\ (\tau_{SR}(c))$. Since $\tau_{SR}(x)$ is a soft rough maximal open set, we also have $SR\ int\ (\tau_{SR}(c)) \subset \tau_{SR}(x)$. Hence, $SR\ int\ \left(\tau_{SR}(c)\right) = \tau_{SR}(x)$.

Theorem 3.12: Let $\tau_{SR}(x)$ be a soft rough maximal open set and $\tau_{SR}(c)$ a non empty soft rough subset of $U - \tau_{SR}(x)$. Then, $U - SR\ cl(\tau_{SR}(c)) = SR\ int\left(U - \tau_{SR}(c)\right) = \tau_{SR}(x)$.

Proof: By our assumption we have $\tau_{SR}(x) \subset U - \tau_{SR}(c) \subsetneq U$. Hence, the proof is immediate by theorem 3.8 and theorem 3.11.

Proposition 3.13: Let $\tau_{SR}(x)$ be a soft rough minimal open set. If $\{x\}$ is an element of $\tau_{SR}(x)$, then $\tau_{SR}(x) \subset \tau_{SR}(n)$ for any soft rough open neighborhood $\tau_{SR}(n)$ of $\{x\}$.

Proof: Let $\tau_{SR}(n)$ be a soft rough open neighborhood of $\{x\}$ such that we have, $\tau_{SR}(x) \not\subset \tau_{SR}(n)$. Then $\tau_{SR}(x) \cap \tau_{SR}(n)$ is a soft rough open set such that $\tau_{SR}(x) \cap \tau_{SR}(n) \subset \tau_{SR}(x)$ and $\tau_{SR}(x) \cap \tau_{SR}(n) \neq \emptyset$, which is a contradiction to our assumption that $\tau_{SR}(x)$ is a soft rough minimal open set.

Proposition 3.14: Let $\tau_{SR}(x)$ be a soft rough minimal open set and $\{x\}$ be an element of U. Then,
Then,$\tau_{SR}(x) = \cap\{\tau_{SR}(n): \tau_{SR}(n) \cap \tau_{SR}(x) \neq \emptyset$,where $\tau_{SR}(n)$ is a soft rough neighborhood of{x}}.

Proof: Since $\tau_{SR}(x)$ is an open neighborhood of $\{x\}$ and by proposition 3.13, $\tau_{SR}(x) \subset \{\tau_{SR}(n): \tau_{SR}(n) \cap \tau_{SR}(x) \neq \emptyset, where\ \tau_{SR}(n)\ is\ an\ soft\ rough\ open\ neighborhood\ of\ \{x\}\} \subset \tau_{SR}(x)$. Thus, $\tau_{SR}(x) = \cap\{\tau_{SR}(n): \tau_{SR}(n) \cap \tau_{SR}(x) \neq \emptyset$,where $\tau_{SR}(n)$ is a soft rough neighborhood of{x}}.

Theorem 3.15: Let $\tau_{SR}(x)$ be a soft rough open set. Then the following conditions are equivalent.

i) $\tau_{SR}(x)$ is soft rough minimal open set.
ii) $\tau_{SR}(x) \subset SR\ cl\ (\tau_{SR}(n))$ for any non empty soft rough subset $\tau_{SR}(n)\ of\ \tau_{SR}(x)$.
iii) $SR\ cl\ (\tau_{SR}(x)) = SR\ cl\ (\tau_{SR}(n))$ for any non empty soft rough subset $\tau_{SR}(n)$ of $\tau_{SR}(x)$.

Proof: (i) ⇒ (ii) Consider $\tau_{SR}(x)$ is a soft rough minimal open set. Let $\tau_{SR}(n)$ be any nonempty soft rough subset of $\tau_{SR}(x)$. By proposition 3.13, we have, $\tau_{SR}(n) = \tau_{SR}(x) \cap \tau_{SR}(h) \subset \tau_{SR}(h) \cap \tau_{SR}(x)$ for any open neighborhood $\tau_{SR}(h)$ of $\{x\} \in \tau_{SR}(x)$. Then, $\tau_{SR}(h) \cap \tau_{SR}(n) \neq \emptyset$ and therefore $\{x\}$ is an element of $SR\ cl\ \tau_{SR}(n)$. That implies $\tau_{SR}(x) \subset SR\ cl(\tau_{SR}(n))$.

(ii) ⇒ (iii) Given, $\tau_{SR}(n)$ is a nonempty soft rough subset of $\tau_{SR}(x)$, ie., $\tau_{SR}(n) \subset \tau_{SR}(x)$.That implies $SR\ cl\ \tau_{SR}(n) \subset SR\ cl\ \tau_{SR}(x)$

We have, $\tau_{SR}(x) \subset SR\ cl\ \tau_{SR}(n) \Rightarrow SR\ cl\ (\tau_{SR}(x)) \subset SR(SR\ cl\ \tau_{SR}(n)) \Rightarrow SR\ cl\ \tau_{SR}(x) \subset SR\ cl\ \tau_{SR}(n)$.

Hence, $SR\ cl\ \tau_{SR}(x) = SR\ cl\ \tau_{SR}(n)$ for any nonempty soft rough subset $\tau_{SR}(n)$ of $\tau_{SR}(x)$.

(iii) ⇒ (i) Suppose $\tau_{SR}(x)$ is not a soft rough minimal open set then there exists a non empty soft rough open set $\tau_{SR}(h)$ such that $\tau_{SR}(h) \subset \tau_{SR}(x)$. Hence, there exists an element $\{x\} \in \tau_{SR}(x)$ such that $\{x\} \notin SR\ cl(\tau_{SR}(h)) \Rightarrow SR\ cl\ \{x\} \subset SR\ cl\ (\tau_{SR}(h))'$ which is contradiction.

4. SOFT ROUGH MINIMAL IRRESOLUTE AND SOFT ROUGH MAXIMAL IRRESOLUTE:

Definition 4.1: Let $(U, \tau_{SR}(X), E)$ and $(V, \tau_{SR}(Y), E)$ be the soft rough topological spaces and a map $f: (U, \tau_{SR}(X), E) \rightarrow (V, \tau_{SR}(Y), E)$ is said to be a soft rough minimal irresolute if $f^{-1}(F(E))$ is a soft rough minimal open set in $(U, \tau_{SR}(X), E)$for a $SR\ Mi\ O(F(E))$in $(V, \tau_{SR}(Y), E)$.

Example 4.2:
$U = \{x_1, x_2, x_3, x_4\}, E = \{e_1, e_2, e_3, e_4\}, A = \{e_1, e_2, e_3\} \subset E$ and S= $\{(e_1, \{x_1, x_2\}), (e_2, \{x_4\}), (e_3, \{x_1, x_3\})\}$. If X⊂U such that $X = \{x_2, x_4\}$, then we have $\underline{R_P}(X) = \{x_4\}$,$\overline{R_P}(X) = \{x_1, x_2, x_4\} and\ Bnd_P(X) = \{x_1, x_2\}$. Thus $\tau_{SR}(X) = \{U, \emptyset, \{x_4\}, \{x_1, x_2, x_4\}, \{x_1, x_2\}\}$ is a soft rough topology. Let $V = \{x_1, x_2, x_3, x_4\}$, $S' = \{(e_1, \{x_1, x_3\}), (e_2, \{x_4\}), (e_3, \{x_2, x_3\})\}$ and $Y = \{x_1, x_4\} \subset V$, then $\underline{R_P}(Y) = \{x_4\}$,$\overline{R_P}(Y) = \{x_1, x_3, x_4\}\ and\ Bnd_P(Y) = \{x_1, x_3\}$. Thus $\tau_{SR'}(Y) = \{V, \emptyset, \{x_4\}, \{x_1, x_3, x_4\}, \{x_1, x_3\}\}$ is a soft rough topology. Define $f: (U, \tau_{SR}(X), E) \rightarrow (V, \tau_{SR}(Y), E)$ is an identity map. Then f is soft rough minimal irresolute map.

Definition 4.3: Let $(U, \tau_{SR}(X), E)$ and $(V, \tau_{SR}(Y), E)$ be the soft rough topological spaces and a map $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ is said to be a soft rough maximal irresolute if $f^{-1}(F(E))$ is a soft rough maximal open set in $(U, \tau_{SR}(X), E)$ for a $SR\ Ma\ O(F(E))$in $(V, \tau_{SR}(Y), E)$.

Example 4.4: $U = \{x_1, x_2, x_3, x_4\}, E = \{e_1, e_2, e_3, e_4\}, A = \{e_1, e_2, e_3\} \subset E$ and S= $\{(e_1, \{x_2\}), (e_2, \{x_1, x_3\}), (e_3, \{x_1, x_4\})\}$. If X⊂U such that $X = \{x_2, x_3\}$, then we have $\underline{R_P}(X) = \{x_2\}$,$\overline{R_P}(X) = \{x_1, x_2, x_3\} and\ Bnd_P(X) = \{x_1, x_3\}$. Thus $\tau_{SR}(X) = \{U, \emptyset, \{x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_3\}\}$ is a soft rough topology. Let $V = \{x_1, x_2, x_3, x_4\}$, $S' = \{(e_1, \{x_3\}), (e_2, \{x_1, x_2\}), (e_3, \{x_2, x_4\})\}$ and $Y = \{x_1, x_3\} \subset V$, then $\underline{R_P}(Y) = \{x_3\}$,$\overline{R_P}(Y) = \{x_1, x_2, x_3\}\ and\ Bnd_P(Y) = \{x_1, x_2\}$. Thus $\tau_{SR'}(Y) = \{V, \emptyset, \{x_3\}, \{x_1, x_2, x_3\}, \{x_1, x_2\}\}$ is a soft rough topology. Define $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ is an identity map. Then f is soft rough maximal irresolute map.

Theorem 4.5: Every soft rough minimal irresolute map is a soft rough minimal continuous map and the converse is not true.

Proof: Consider $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ is a soft rough minimal irresolute map. By the definition of 4.1, $f^{-1}(SR\ Mi\ O(\tau_{SR}(Y))$ is a soft rough minimal open set in $(U, \tau_{SR}(X), E)$ for every soft rough minimal open set $SR\ Mi\ O(\tau_{SR}(Y))$in $(V, \tau_{SR}(Y), E)$. Since, every soft rough minimal open set is soft rough open set implies, $f^{-1}(SR\ Mi\ O(\tau_{SR}(Y))$ is a soft rough open set. Thus $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ is a soft rough minimal continuous. Converse is obvious by every soft rough open set does not imply soft rough minimal open set.

Example 4.6: $U = \{x_1, x_2, x_3, x_4\}, E = \{e_1, e_2, e_3, e_4\}, A = \{e_1, e_2, e_3\} \subset E$ and = $\{(e_1, \{x_1, x_2\}), (e_2, \{x_2\}), (e_3, \{x_3, x_4\})\}$. If $X \subset U$ such that $X = \{x_2\}$, then we have $\underline{R_P}(X) = \{x_2\}$,$\overline{R_P}(X) = \{x_1, x_2\} and\ Bnd_P(X) = \{x_1\}$. Thus $\tau_{SR}(X) = \{U, \emptyset, \{x_2\}, \{x_1, x_2\}, \{x_1\}\}$ is a soft rough topology. Let $V = \{x_1, x_2, x_3, x_4\}$, $S' = \{(e_1, \{x_1, x_2\}), (e_2, \{x_1, x_3\}), (e_3, \{x_1, x_4\})\}$ and $Y = \{x_1, x_2\} \subset V$, then $\underline{R_P}(Y) = \{x_1, x_2\}$,$\overline{R_P}(Y) = \{x_1, x_2, x_3, x_4\}\ and\ Bnd_P(Y) = \{x_3, x_4\}$. Thus $\tau_{SR'}(Y) = \{V, \emptyset, \{x_1, x_2\}, \{x_1, x_2, x_3, x_4\}, \{x_3, x_4\}\}$ is a soft rough topology. Define $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ is an identity map. Then f is soft rough minimal continuous but not soft rough minimal irresolute map.

Theorem 4.7: Every soft rough maximal irresolute map is soft rough maximal continuous map and the converse is not true.

Proof: Similar to theorem 4.6.

Remark 4.8: Soft rough minimal irresolute maps are independent of soft rough continuous (resp. soft rough maximal continuous) map.

Example 4.9: $U = \{x_1, x_2, x_3, x_4\}, E = \{e_1, e_2, e_3, e_4\}, A = \{e_1, e_2, e_3\} \subset E$ and $S = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2\}), (e_3, \{x_3, x_4\})\}$. If $X \subset U$ such that $X = \{x_1, x_2\}$, then we have

$\underline{R_P}(X) = \{x_2\}$,$\overline{R_P}(X) = \{x_1, x_2, x_3\} and\ Bnd_P(X) = \{x_1, x_3\}$. Thus $\tau_{SR}(X) = \{U, \emptyset, \{x_2\}, \{x_1, x_2, x_3\}, \{x_1, x_3\}\}$ is a soft rough topology. Let $V = \{x_1, x_2, x_3, x_4\}$, $S' = \{(e_1, \{x_1, x_3\}), (e_2, \{x_2\}), (e_3, \{x_1, x_4\})\}$ and $Y = \{x_2, x_4\} \subset V$, then $\underline{R_P}(Y) = \{x_2\}$,$\overline{R_P}(Y) = \{x_1, x_2, x_4\}\ and\ Bnd_P(Y) = \{x_1, x_4\}$. Thus $\tau_{SR'}(Y) = \{V, \emptyset, \{x_2\}, \{x_1, x_2, x_4\}, \{x_1, x_4\}\}$ is a soft rough topology. Define $f: (U, \tau_{SR}(X), E) \to (V, \tau_{SR}(Y), E)$ is an identity map. Here f is soft rough minimal irresolute but not soft rough continuous.

Remark 4.10: Soft rough maximal irresolute maps are independent of sort rough continuous (resp. soft rough minimal continuous map).

Example 4.11: $U = \{x_1, x_2, x_3, x_4\}, E = \{e_1, e_2, e_3, e_4\}, A = \{e_1, e_2, e_3\} \subset E$

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$S=\{(e_1,\{x_1,x_2\}),(e_2,\{x_4\}),(e_3,\{x_2,x_3\})\}$. If $X\subset U$ such that $X=\{x_1,x_2\}$,then we have $\underline{R_P}(X)=\{x_1,x_2\}$,$\overline{R_P}(X)=\{x_1,x_2,x_3\} and\ Bnd_P(X)=\{x_3\}$. Thus $\tau_{SR}(X)=\{U,\emptyset,\{x_1,x_2\},\{x_1,x_2,x_3\},\{x_3\}\}$ is a soft rough topology. Let $V=\{x_1,x_2,x_3,x_4\}$, $S'=\{(e_1,\{x_1,x_3\}),(e_2,\{x_2,x_3\}),(e_3,\{x_4\})\}$ and $Y=\{x_1,x_3\}\subset V$, then we have $\underline{R_P}(Y)=\{x_1,x_3\}$, $\overline{R_P}(Y)=\{x_1,x_2,x_3\}\ and\ Bnd_P(Y)=\{x_2\}$. Thus $\tau_{SR'}(Y)=\{V,\emptyset,\{x_1,x_3\},\{x_1,x_2,x_3\},\{x_2\}\}$ is a soft rough topology.
Define $f:(U,\tau_{SR}(X),E)\to(V,\tau_{SR}(Y),E)$ be an identity map. Here f is soft rough maximal irresolute but it is not a soft rough continuous.

Remark 4.12: soft rough maximal irresolute and soft rough minimal irresolute maps are independent.

Theorem 4.13: consider $(U,\tau_{SR}(X),E)$ and $(V,\tau_{SR}(Y),E)$ are soft rough topological spaces and a map $f:(U,\tau_{SR}(X),E)\to(V,\tau_{SR}(Y),E)$ is a soft rough minimal irresolute if and only if inverse image of every soft rough maximal closed set in $(V,\tau_{SR}(Y),E)$ is a soft rough maximal closed set in $(U,\tau_{SR}(X),E)$.

Proof: The proof follows from the definition and with the fact that complement of soft rough minimal open set is soft rough maximal closed set.

Theorem 4.14: Composition of soft rough minimal irresolute map is soft rough minimal irresolute map.

Proof: Consider $f:(U,\tau_{SR}(X),E)\to(V,\tau_{SR}(Y),E)$ and $g:(V,\tau_{SR}(Y),E)\to(W,\tau_{SR}(Z),E)$ are soft rough irresolute map. Consider a soft rough minimal open set $SR\ Mi\ O(\tau_{SR}(Z))$ in $(W,\tau_{SR}(Z),E)$. By definition of soft rough minimal irresolute map, $g^{-1}(SR\ Mi\ O(\tau_{SR}(Z))$ is a soft rough minimal open set in $(V,\tau_{SR}(Y),E)$. $f^{-1}(g^{-1}(SR\ Mi\ O(\tau_{SR}(Z))$is also a soft rough minimal open set in $(U,\tau_{SR}(X),E)$ since f is a soft rough minimal irresolute. Therefore, $f^{-1}(g^{-1}(SR\ Mi\ O(\tau_{SR}(Z))=(g\circ f)^{-1}SR\ Mi\ O(\tau_{SR}(Z))$. Thus, $g\circ f$ is a soft rough minimal irresolute.

Theorem 4.15: consider $(U,\tau_{SR}(X),E)$ and $(V,\tau_{SR}(Y),E)$ are soft rough topological spaces and a map $f:(U,\tau_{SR}(X),E)\to(V,\tau_{SR}(Y),E)$ is a soft rough maximal irresolute if and only if inverse image of every soft rough minimal closed set in $(V,\tau_{SR}(Y),E)$ is a soft rough minimal closed set in $(U,\tau_{SR}(X),E)$.

Proof: The proof follows from the definition and with the fact that complement of soft rough maximal open set is soft rough minimal closed set.

Result 4.16: Composition of soft rough maximal irresolute map is soft rough maximal irresolute map.

REFERENCES

[1] Z. Pawlak, Rough sets. International Journal of Computer and Information Sciences (1982) 11 341-356.
[2] D. Molodtsov, Soft Set Theory-first results, Computers and Mathematics with Applications 37 (1999) 19-31.
[3] Feng Feng, Xiaoyan Lin, Violeta Leoreanu-Fotea and Young Bae Jun., Soft sets and soft rough sets. Information Sciences 181 (2011) 1125-1137.
[4] L. A. Zadeh, Fuzzy sets, Information and control 8 (1965) 338-353.
[5] Z. Pawlak, Rough sets: Theoretical Aspects of Reasoning about Data, Kluwer Academic Publishers, Boston, 1991.
[6] D. Molodtsov, The Theory of Soft sets, USSR Publishers, Moscow, 2004 (in Russian).
[7] P. K. Maji, R.Biswas, A.R.Roy, Soft set theory, Computers and Mathematics with Applications 45 (2003) 555-562.
[8] H. Aktas, N. Cagman, Soft sets and soft groups, Information Sciences 177 (2007) 2726-2735.
[9] M. Lellis Thivagar, Carmel Richard, Nirmala Rebecca Paul, Mathematical Innovations of a Modern Topology in Medical Events, International Journals of Information Science 2012, 2(4): 33-36.
[10] Muhammad Shabir*, Munazza Naz, On Soft Topological Spaces, Computers and Mathematics with Applications 61 (2001) 1786-1799.

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[11] F. Feng, C. X. Li, B. Davvaz, M.I. Ali, Soft sets combined with fuzzy sets and rough sets: a tentative approach, Soft Computing 14 (2010) 899-911.
[12] M. Y. BAKIER, A. A. ALLAM, SH. S. ABD-ALLAH; Soft Rough Topology, Annals of Fuzzy Mathematics and Informatics, Volume, 11 No. 2, (February 2016), pp, 4-xx.
[13] F. Nakaoka and N. Oda, some applications of minimal open sets, $Int.J.$Math. Math. Sci. 27-8, 471-476(2001)
[14] F. Nakaoka and N. Oda, some properties of maximal open sets, $Int.J.$Math. Math. Sci. 21, 1331-1340(2003)
[15] F. Nakaoka and N. Oda, On minimal closed sets, proceeding of Topological spaces Theory and its Applications, 5, 19-21(2003).
[16] S. S. BENCHALLI, BASAVARAJ M.ITTANAGI and R.S. WALI. On Minimal Open Sets and Maps in Topological Spaces, J. Comp. & Math. Sci. Vol. 2 (2) 208-220 (2001).
[17] Qays Shakir,[a,b] Minimal and Maximal Soft Open Sets, an international journal of mathematical sciences with computer applications, Malaya J. Mat. 4(1) (2016) 53-58.
[18] K.Uthra,K. Bhuvaneswari, J. Sheeba Priyadharshini, Rough generalized minimal and maximal closed sets, International journals in Management and Social science. Volume 07 Issue 03, march 2019 ISSN: 2321-1784.
[19] J. Rajalakshmi and R. selvi, "A study on soft rough minimal continuous and soft rough maximal continuous", International Journal of Mathematics and Computational sciences-Accepted